Section Two: Calculator-assumed **65% (99 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

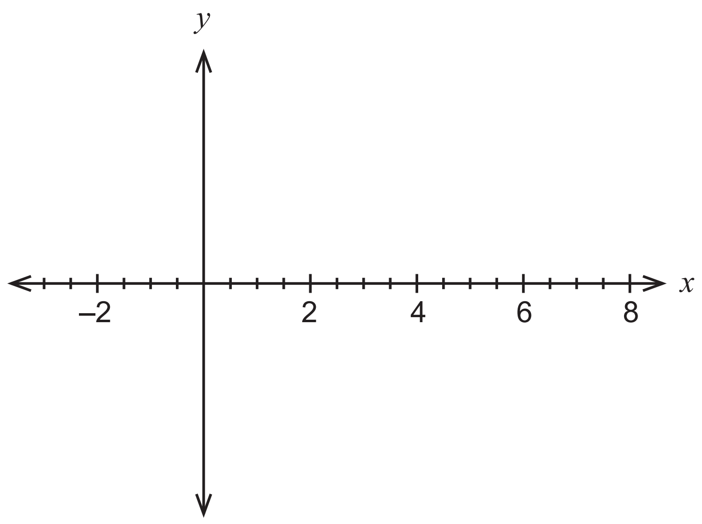
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 2018 WACE **(8 marks)**

Consider the function $f(x) = \log_a(x - 1)$ where $a > 1$.

(a) On the axes below, sketch the graph of $f(x)$, labelling important features. (3 marks)



(b) Determine the value of m if $f(m) = 1$. (2 marks)

(c) Determine the coordinates of the x – intercept of $f(x + b) + c$, where b and c are positive real constants. (3 marks)

Question 18 2018 WACE **(7 marks)**

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, L, is given by the formula below:

$L = 10\log\left(\frac{I}{I_0}\right)$ dB where I is the sound intensity and I_0 is the reference sound intensity.

I and I_0 are measured in watt/m^2.

(a) Listening to a sound intensity of 5 billion times that of the reference intensity $(I = 5 \times 10^9 I_0)$ for more than 30 minutes is considered unsafe. To what sound intensity level does this correspond? (2 marks)

(b) The reference sound intensity, I_0, has a sound intensity level of 0 dB. If a household vacuum cleaner has a sound intensity $I = 1 \times 10^{-5}$ watt/m^2 and this corresponds to a sound intensity level $L = 70$ dB, determine I_0. (2 marks)

The average sound intensity level for rainfall is 50 dB and for heavy traffic 85 dB.

(c) How many times more intense is the sound of traffic than that of rainfall? (3 marks)

End of questions

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 7 2017 WACE **(6 marks)**

Given that $\log_{10}2 = x$ and $\log_{10}7 = y$

(a) express $\log_{10}14$ in terms of x and y. (2 marks)

(b) show that $\log_{10}17.5 = y - 2x + 1$. (2 marks)

(c) evaluate 10^{y-x}. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free 2016 WACE **35% (49 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

Working time: 50 minutes.

Question 1 **(5 marks)**

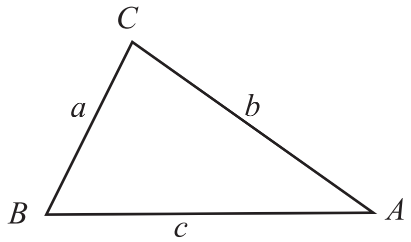
(a) Given that $\log_8 x = 2$ and $\log_2 y = 5$, evaluate $x - y$. (2 marks)

(b) Express y in terms of x given that $\log_2 (x + y) + 2 = \log_2 (x - 2y)$. (3 marks)

NOT ON LOGS

Question 11 **(3 marks)**

The area of a triangle can be found by the formula: $Area = \dfrac{ab \sin C}{2}$.



Using the incremental formula, determine the approximate change in area of an equilateral triangle, with each side of 10 cm, when each side increases by 0.1 cm.

Question 12 2016 WACE **(3 marks)**

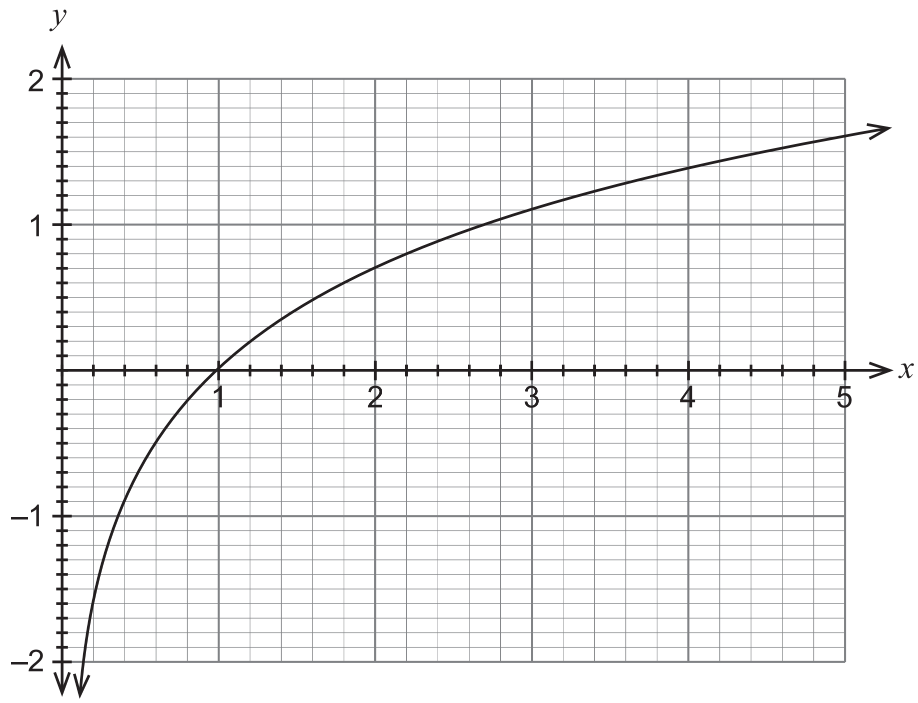
The Richter magnitude, M, of an earthquake is determined from the logarithm of the amplitude, A, of waves recorded by seismographs.

$M = \log_{10} \dfrac{A}{A_o}$, where A_o is a reference value.

An earthquake in a town in New Zealand in November 2015 was estimated at 5.5 on the Richter scale, while the earthquake just north of Hayman Island measured 3.4 on the same scale. How many times larger was the amplitude of the waves in New Zealand compared to those at Hayman Island?

Question 4 **(6 marks)**

Consider the graph of $y = \ln(x)$ shown below.

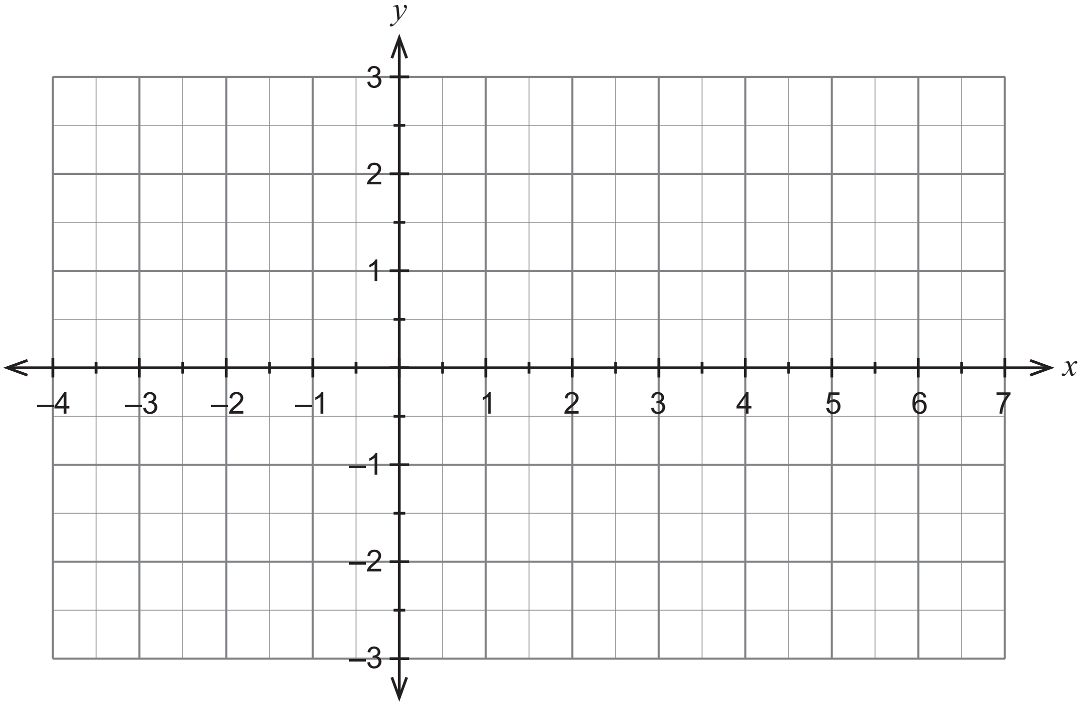


(a) Use the graph to estimate the value of p in each of the following.

(i) $1.4 = \ln(p)$ (1 mark)

(ii) $e^{p+1} - 3 = 0$ (2 marks)

(b) On the axes below, sketch the graph of $y = \ln(x - 2) + 1$. (3 marks)

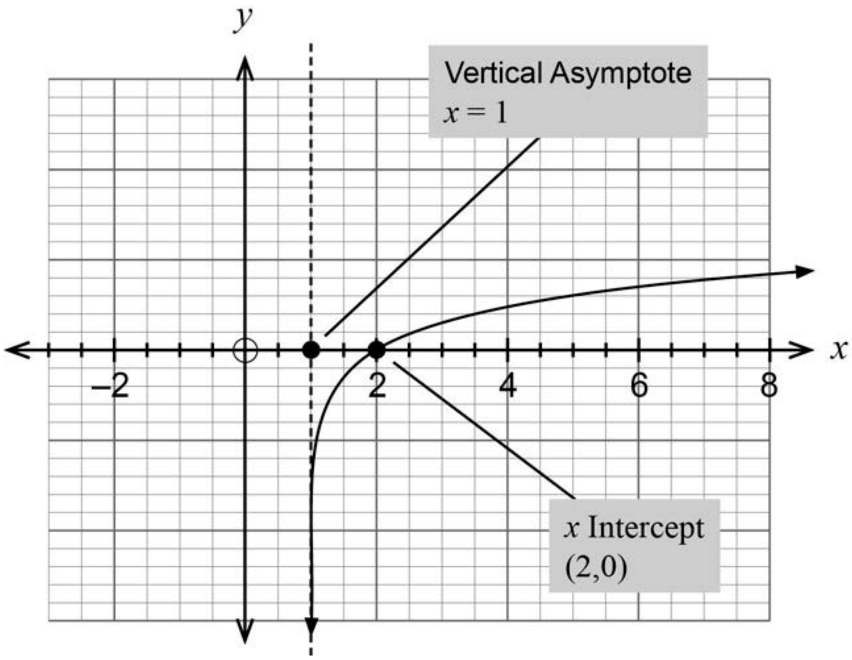


Section Two: Calculator-assumed **65% (99 Marks)**

Question 8 **(8 marks)**

Consider the function $f(x) = \log_a(x-1)$ where $a > 1$.

(a) On the axes below, sketch the graph of $f(x)$, labelling important features. (3 marks)



Solution
See graph
Specific behaviours
✓ asymptote at $x = 1$ ✓ gives correct shape ✓ x-int at $x = 2$

(b) Determine the value of m if $f(m) = 1$. (2 marks)

Solution
$1 = \log_a(m-1)$ $m-1 = a$ $m = a+1$
Specific behaviours
✓ equates $f(m)$ to 1 ✓ solves for m

(c) Determine the coordinates of the x – intercept of $f(x+b)+c$, where b and c are positive real constants. (3 marks)

Solution
$0=\log_a(x-1+b)+c$ $-c=\log_a(x-1+b)$ $a^{-c}=x-1+b$ $x=a^{-c}+1-b$ coordinates are: $\left(a^{-c}+1-b,0\right)$
Specific behaviours
✓ equates new function to zero ✓ solves for x ✓ states coordinates

Question 18 **(7 marks)**

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, L, is given by the formula below:

$L = 10\log\left(\frac{I}{I_0}\right)$ dB where I is the sound intensity and I_0 is the reference sound intensity.

I and I_0 are measured in watt/m^2.

(a) Listening to a sound intensity of 5 billion times that of the reference intensity ($I = 5\times10^9 I_0$) for more than 30 minutes is considered unsafe. To what sound intensity level does this correspond? (2 marks)

Solution
$L = 10\log\left(\frac{5\times10^9 I_0}{I_0}\right)$ ≈ 97 dB
Specific behaviours
✓ substitutes for L ✓ calculates level

(b) The reference sound intensity, I_0, has a sound intensity level of 0 dB. If a household vacuum cleaner has a sound intensity, $I = 1\times10^{-5}$ watt/m^2 and this corresponds to a sound intensity level $L = 70$ dB, determine I_0. (2 marks)

Solution
$70 = 10\log\left(\frac{1\times10^{-5}}{I_0}\right)$ $I_0 = \frac{1\times10^{-5}}{10^7} = 1\times10^{-12}$ watt/m^2
Specific behaviours
✓ substitutes for L and I ✓ determines I_0 including units

The average sound intensity level for rainfall is 50 dB and for heavy traffic 85 dB.

(c) How many times more intense is the sound of traffic than that of rainfall? (3 marks)

Solution
$50 = 10\log\left(\frac{I_{rain}}{I_0}\right) \Rightarrow \frac{I_{rain}}{I_0} = 10^5 \Rightarrow I_{rain} = 10^5 I_0$ $85 = 10\log\left(\frac{I_{traffic}}{I_0}\right) \Rightarrow \frac{I_{traffic}}{I_0} = 10^{8.5} \Rightarrow I_{traffic} = 10^{8.5} I_0$ $\therefore \frac{I_{traffic}}{I_{rain}} = \frac{10^{8.5}}{10^5} = 10^{3.5} \approx 3200$
Specific behaviours
✓ rearranges logarithmic equations to exponentials ✓ writes ratio and cancels I_0 ✓ determines how many more times intense

Question 7 **(6 marks)**

Given that $\log_{10} 2 = x$ and $\log_{10} 7 = y$

(a) express $\log_{10} 14$ in terms of x and y. (2 marks)

Solution
$\log_{10} 14 = \log_{10}(2 \times 7) = \log_{10}2 + \log_{10}7 = \text{x} + \text{y}$
Specific behaviours
✓ expresses 14 as 2×7 ✓ uses log laws to obtain the expression

(b) show that $\log_{10} 17.5 = y - 2x + 1$. (2 marks)

Solution
$\log_{10}17.5 = \log_{10}\frac{70}{4} = \log_{10}7 + \log_{10}10 - \log_{10}2^2 = y - 2x + 1$
Specific behaviours
✓ uses log laws correctly to expand ✓ uses the log law for a power to obtain the correct expression

(c) evaluate 10^{y-x}. (2 marks)

Solution
$10^x = 2$ $10^y = 7$ $10^{y-x} = \frac{10^y}{10^x}$ $= \frac{7}{2}$
Specific behaviours
✓ rewrites logarithmic equations in exponential form ✓ uses index laws to evaluate

Question 1 **(5 marks)**

(a) Given that $\log_8 x = 2$ and $\log_2 y = 5$, evaluate $x - y$. (2 marks)

Solution
$8^2 - 2^5 = 32$
Specific behaviours
✓ determines x and y ✓ recognises the inverse relationship between logarithms and exponentials

(b) Express y in terms of x given that $\log_2(x+y) + 2 = \log_2(x-2y)$. (3 marks)

Solution
$\log_2(x+y)+2=\log_2(x-2y)$ $\log_2(x+y)+\log_2 4=\log_2(x-2y)$ $\log_2(4(x+y))=\log_2(x-2y)$ $4(x+y)=(x-2y)$ $4x+4y=x-2y$ $6y=-3x$ $y=\frac{-1}{2}x$
Specific behaviours
✓ expresses all terms as logarithms ✓ uses log laws to combine terms ✓ expresses y in terms of x

Question 12 **(3 marks)**

The Richter magnitude, M, of an earthquake is determined from the logarithm of the amplitude, A, of waves recorded by seismographs.

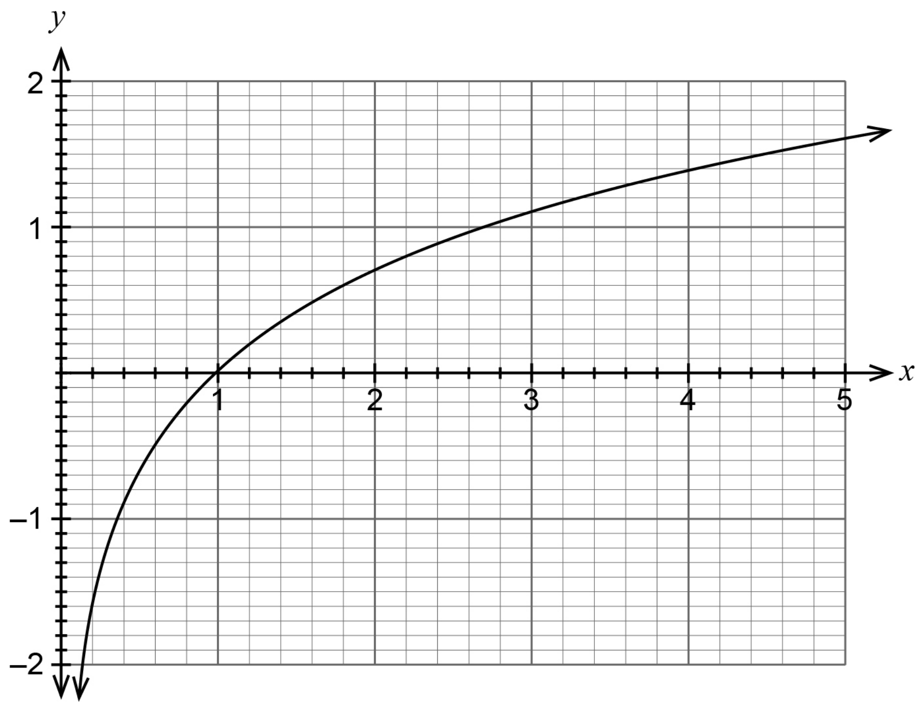
$M = \log_{10}\frac{A}{A_o}$, where A_o is a reference value.

An earthquake in a town in New Zealand in November 2015 was estimated at 5.5 on the Richter scale, while the earthquake just north of Hayman Island measured 3.4 on the same scale. How many times larger was the amplitude of the waves in New Zealand compared to those at Hayman Island?

Solution
$M = \log_{10}\frac{A}{A_o}$ $A = A_o 10^M$ $\frac{A_{NZ}}{A_H} = \frac{10^{5.5}}{10^{3.4}} = 10^{2.1}$
Specific behaviours
✓ converts log statement to an index form ✓ subtracts Richter magnitudes ✓ determines ratio of amplitudes

Question 4 **(6 marks)**

Consider the graph of $y = \ln(x)$ shown below.



(a) Use the graph to estimate the value of p in each of the following.

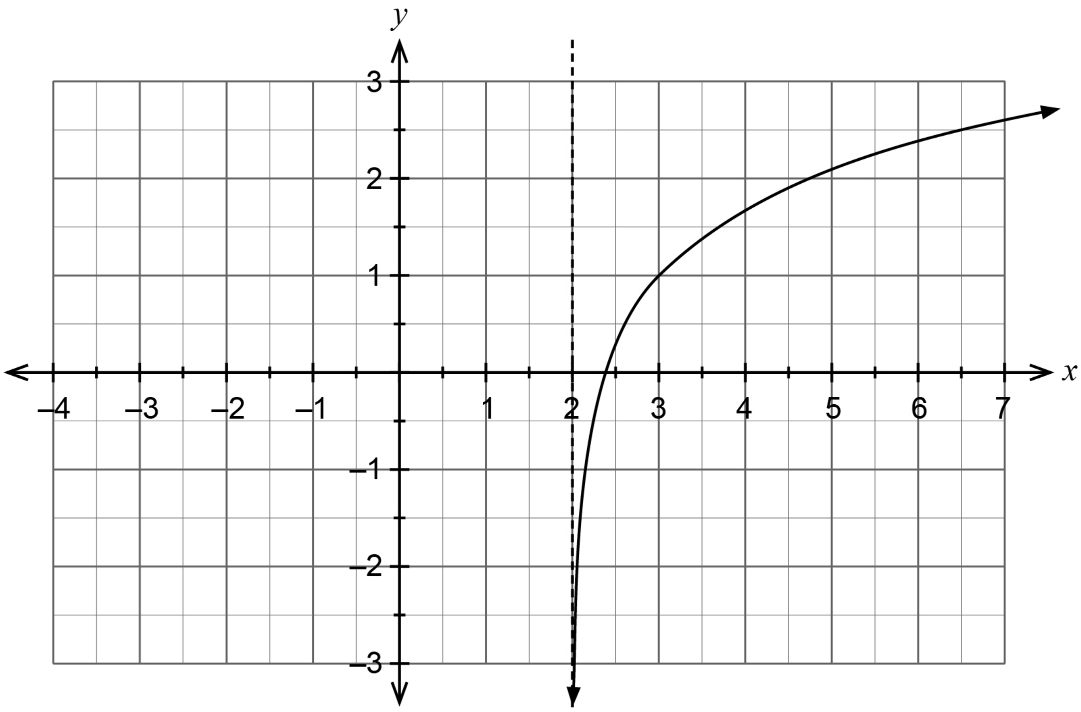
(i) $1.4 = \ln(p)$ (1 mark)

Solution
$p = 4$
Specific behaviours
✓ states the correct value of p

(ii) $e^{p+1} - 3 = 0$ (2 marks)

Solution
$e^{p+1} = 3$ $p + 1 = \ln(3)$ $p + 1 = 1.1$ $\therefore p = 0.1$
Specific behaviours
✓ rearranges to form a logarithmic equation ✓ states the correct value of p

(b) On the axes below, sketch the graph of $y=\ln(x-2)+1$. (3 marks)



Solution
Specific behaviours
✓ draws asymptote at $x=2$ ✓ the sketch passes through the point (3,1) ✓ the sketch has the correct shape and has a y-coordinate between 2.5 and 3 when $x=7$